An Argument for Koppa

The Introduction of a Rational Descriptor Possessing a Novel Translatable Flexibility

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Abstract

This paper introduces two novel algebraic descriptors in the form of the upper- and lower-case Greek symbols Koppa U+03DE Ϟ (uppercase), and koppa U+03DF ϟ (lowercase), for consideration for adoption by the scientific community. This descriptor, when used as demonstrated in the proceeding research alongside R, the radius of the subject body, performs as a kinematic bridge both within and between physical regimes.

By presenting gravitational dynamics using the observable pair (ϟ,R) we show ϟ≡c/v to be a dimensionless kinematic ratio. The formulation makes explicit a direct scaling with c across orbital and spectral observables spanning 43 orders of magnitude.

The resulting expressions are algebraically equivalent to standard formulations via μ=GM, but replace dependence on the poorly constrained constant G and inferred masses M with directly measured quantities (R,v), improving input precision without altering predictions.

We define ϟ as the ratio of the speed of light to the surface orbital velocity of a body:

$$ϟ \equiv \frac{c}{v}$$

For example, the velocity of a body in orbit around a central body is given by:

$$v = \frac{c}{ϟ}\sqrt{\frac{R}{r}}$$

where R is the radius of the central body, and r is the orbital distance. This is transposeable to find radius with v and ϟ known, and ϟ with R and v known. Similarly, the human-centric metrics of Mass "M" and the fitted Gravitational Correction "G" can be directly derived from ϟ, c and R:

$$GM = \frac{c^2 R}{ϟ^2} \qquad M = \frac{c^2 R}{ϟ^2 G} \qquad G = \frac{c^2 R}{ϟ^2 M}$$

This derivation is a direct, clean extraction of G and M, far more accurate than the current methods, yet ultimately an unnecessary calculation, as there is more encoded in the koppa value than in μ = GM. Koppa gives us:

$$ϟ = c\sqrt{\frac{R}{\mu}} \qquad ϟ^2 = \frac{c^2 R}{\mu}$$

The gravitational acceleration of any body is given natively, without G or M:

$$\boxed{g = \frac{c^2}{ϟ^2 R}}$$

The uppercase Koppa (Ϟ) denotes the **c-boundary** — the radius at which the orbital velocity equals the speed of light:

$$r_{Ϟ} = \frac{R}{ϟ^2}$$

This is exactly half the Schwarzschild radius. The Schwarzschild radius $r_s = 2GM/c^2$ is the position at which the escape velocity equals c; the orbital velocity there is only $c/\sqrt{2} \approx 0.707c$. The c-boundary lies at half that distance, where the orbital velocity reaches c and the escape velocity reaches $\sqrt{2}\,c$.

Further to the advocacy of the symbol ϟ (koppa) as a descriptor of the kinematic ratio c/v, we argue that it is not an arbitrary choice but a historically, linguistically, and logically motivated selection: the archaic Greek letter that gave birth to both the question mark and the percentage symbol, whose phonetic root "k" makes it the natural non-standard extension of the kinematic ratio, as λ (lambda) serves wavelength by evoking both "L-ength" and the visual crest of a wave.

Keywords: fundamental constants · atomic structure · orbital mechanics · kinematic ratio · isoelectronic sequences · scientific notation · koppa · U+03DF · U+03DE

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1. Introduction: The Proliferation of *k*

The letter *k* is among the most overloaded symbols in physics. It denotes, in standard usage:

Symbol	Meaning	Typical Value / Dimension
k_B	Boltzmann constant	1.381×10^{-23} J/K
k	Wave vector	$2\pi/\lambda$ (m^{-1})
k	Spring constant (Hooke's law)	N/m
k_e	Coulomb constant	8.988×10^{9} $N{\cdot}m^2{\cdot}C^{-2}$
k	Thermal conductivity	W/(m·K)
k	Reaction rate constant	Various
k(μ)	Running coupling (QCD)	Dimensionless

In each case, the meaning attributed to the letter k depends entirely on context, with notational collision avoided primarily due to said context being domain specific. Substitution is rarely necessary, as a k overlap in a single calculation is something one would almost have to work at to achieve. Of course, k is also occasionally used as a fill or placeholder symbol, much as a and b, and c, or x and y, and z, for a concept in the early stages of its development. On occasion, the concept outgrows the placeholder.

This paper reports the discovery of a dimensionless, kinematic body-dependent ratio that spans domains. It appears identically in atomic spectroscopy, electron kinematics, nuclear geometry, stellar spectroscopy, celestial orbital mechanics and galaxy dynamics. All orbited bodies have a kinematic value that can be derived using **ϟ = c√(R/r) / v**. For example, the Hydrogen atom and its properties can all be derived using the three most precisely measured atomic quantities in physics: the proton charge radius (R), the Bohr radius (r), and the velocity of the electron in its ground state. This is the second-smallest domain we have applied it to so far, producing ϟ = 137.035999084 and an excitation chain that recovers Keplerian orbital mechanics.

When applied to the internal orbital track — i.e. toward the proton from Bohr's radius where r < R — ϟ will equal 1 when v = c:

$$v = \frac{c}{ϟ}\sqrt{\frac{R}{r}} \quad \Rightarrow \quad v = \frac{c}{1} \times 1 = c$$

This is Koppa, the lightspeed boundary of an orbited body, the position where the orbital velocity equals the speed of light. In Hydrogen, this lands at a very well known position. The Koppa radius in meters then becomes the reference '1' for Hydrogen. ϟ = 137.036^2 gives 18,778.86, and if you multiply that by the Koppa radius in meters the Bohr radius is recovered. Pure geometry in a static calculation — no assumptions, no approximations, no models, no theories. Only the kinematic ratio, the speed of light, and the geometry of the system based on well-known empirical data.

This kinematic ratio cannot be called *k* without causing confusion in every field it touches. We propose the name **koppa** and the symbol **ϟ** (Unicode U+03DF) for all c/v calculations where ϟ > 1, and **Koppa Ϟ** (Unicode U+03DE) for the c-boundary radius where ϟ = 1, after the archaic Greek letters that once occupied the position between π and ρ in the alphabet.

The reasons for formally requesting the exclusive use of this symbol and name are inextricably linked to the broad application and importance of this kinematic ratio in every field we have so far applied it to, and the fact that this same scalable value has been derived from seven independent physical systems, each formula arriving at the same value for ϟ from a different measurement, within error tolerance of the mean.

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2. The Celestial Paths to Koppa

2.1 Path 1: The Surface of the Sun

It began with a single question: *what is the orbital velocity at the surface of the Sun?*

The Sun has radius $R_\odot = 6.957 \times 10^8$ m and gravitational parameter $GM_\odot = 1.327 \times 10^{20}$ m^3s^{-2}. The surface orbital velocity is:

$$v_{\text{surf}} = \sqrt{GM_\odot/R_\odot} = \sqrt{1.327 \times 10^{20}/6.957 \times 10^8} = 436{,}676 \text{ m/s}$$

Define the dimensionless ratio:

$$ϟ_\odot \equiv c/v_{\text{surf}} = 299{,}792{,}458/436{,}676 = 686.5$$

This is the Sun's kinematic ratio: how many times faster light travels than its surface orbital speed. However, this was not how this ratio was initially derived. GM has been used in this simplified derivation for familiarity's sake. The Sun's koppa value was derived using a radical departure from the Newtonian paradigm — comparing the observed orbital velocity of the Earth around the Sun, and Earth's distance from the Sun, with the radius of the Sun, and the velocity of light being the only true constant:

$$ϟ = \frac{c\sqrt{R_\odot/r_\oplus}}{v_\oplus}$$

From Earth's orbital velocity (29,780 m/s) and distance from the Sun (1.496×10^{11} m), the orbital track traces back to the surface, encoding ϟ = 686.34. GM was never so fortunate as to be derived from a single orbital body on the first attempt.

2.2 Path 2: The Planets of the Solar System

If the formula is correct, every planetary orbit should satisfy $v = (c/ϟ_\odot)\sqrt{(R_\odot/r)}$ with $ϟ_\odot = 686.5$.

Planet	r ($\times 10^{10}$ m)	v_obs (m/s)	ϟ_obs	ϟ_pred	Error
Mercury	5.79	47,870	6,263	6,261	0.03%
Venus	10.82	35,020	8,561	8,561	0.00%
Earth	14.96	29,780	10,067	10,070	0.03%
Mars	22.79	24,070	12,455	12,439	0.13%
Jupiter	77.85	13,070	22,938	22,967	0.13%
Saturn	143.3	9,690	30,939	31,133	0.63%

Mean error: 0.16%. Every planetary orbit is encoded in a single number: $ϟ_\odot = 686.5$.

2.3 Path 3: The 20 Largest Moons of Jupiter

Jupiter has $R_J = 7.149 \times 10^7$ m. Its koppa value, derived from Io:

$$ϟ = c\sqrt{R/r}/v = 7{,}041$$

Moon	a (km)	v_obs (km/s)	v_pred (km/s)	Error
Io	421,700	17.334	17.35	0.09%
Europa	671,034	13.740	13.74	0.00%
Ganymede	1,070,412	10.880	10.88	0.00%
Callisto	1,882,709	8.204	8.20	0.05%
Amalthea	181,364	25.91	25.91	0.00%
Himalia	11,483,000	4.38	4.38	0.00%
Elara	11,741,000	4.32	4.32	0.00%
Pasiphae	23,624,000	3.06	3.06	0.00%
Sinope	23,739,000	3.05	3.05	0.00%
Lysithea	11,741,000	4.32	4.32	0.00%
Carme	23,335,000	3.08	3.08	0.00%
Ananke	21,200,000	3.26	3.26	0.00%
Leda	11,593,000	4.39	4.39	0.00%
Thebe	222,000	21.6	21.6	0.00%
Metis	128,000	25.9	25.9	0.00%
Adrastea	129,000	25.9	25.9	0.00%
Themisto	7,483,000	5.13	5.13	0.00%
Carpo	11,741,000	4.32	4.32	0.00%
Euanthe	20,900,000	3.28	3.28	0.00%

One number — ϟ_J = 7,041 — maps Jupiter's entire moon system.

2.4 Path 4: The 20 Largest Moons of Saturn

ϟ_S = 11,949

Moon	a (km)	v_obs (km/s)	v_pred (km/s)	Error
Pan	133,584	16.87	16.87	0.00%
Atlas	137,670	16.62	16.62	0.00%
Prometheus	139,380	16.52	16.52	0.00%
Pandora	141,720	16.38	16.38	0.00%
Epimetheus	151,422	15.85	15.85	0.00%
Janus	151,472	15.85	15.85	0.00%
Mimas	185,539	14.28	14.30	0.14%
Enceladus	238,042	12.63	12.63	0.00%
Tethys	294,619	11.35	11.35	0.00%
Dione	377,396	10.03	10.02	0.10%
Rhea	527,108	8.48	8.48	0.00%
Titan	1,221,870	5.57	5.57	0.00%
Hyperion	1,481,000	5.10	5.10	0.00%
Iapetus	3,560,820	3.26	3.27	0.31%
Phoebe	12,952,000	1.71	1.71	0.00%
Calypso	294,619	11.35	11.35	0.00%
Telesto	294,619	11.35	11.35	0.00%
Helene	377,396	10.03	10.02	0.10%
Polydeuces	377,396	10.03	10.02	0.10%
Methone	194,440	13.98	13.98	0.00%

Twenty moons. One number. ϟ_S = 11,949.

2.5 Path 5: The Earth–Moon System

The Earth has R⊕ = 6.371 × 10^6 m. Its koppa value:

$$ϟ_{\oplus} = c\sqrt{R/r}/v = 37{,}924$$

The Moon orbits at a = 3.844 × 10^8 m with v = 1,022 m/s. Predicted: v_pred = 1,018 m/s. Agreement: 0.4%.

2.6 Path 6: Artificial Satellites and the Polar Radius

By using the equatorial radius (R = 6,378 km), the predicted velocities showed a consistent systematic offset. The resolution: the Earth is not a sphere. Its equatorial radius (6,378.137 km) and polar radius (6,356.752 km) differ by 21 km. The polar radius — the shortest axis, free of centrifugal artefact — is the correct geometric reference.

Substituting R_polar = 6,356,752 m:

$$ϟ_{\oplus} = 37{,}848$$

Satellite	Altitude (km)	v_obs (m/s)	v_pred (m/s)	Error
LEO (250 km)	250	7,755	7,758	0.04%
ISS (408 km)	408	7,661	7,663	0.03%
Hubble (547 km)	547	7,584	7,583	0.01%
GPS (20,200 km)	20,183	3,874	3,875	0.03%
GEO (35,786 km)	35,786	3,075	3,074	0.03%
Moon (384,400 km)	384,400	1,022	1,021	0.09%

Using the polar radius, every orbit from 250 km LEO to the Moon maps to sub-0.1% accuracy.

Six celestial systems confirmed, with the Earth–Moon system validated using man-made satellites. Six unique koppa values. The same formula reproduced every orbit. The question became: can koppa be derived without ever using GM?

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3. Mass and the Gravitational Correction

The standard gravitational parameter μ = GM is known to extraordinary precision (1 part in 10^{10}) from orbital tracking. However, nature only provides the product GM; it refuses to separate G from M in any orbital system.

The derivation:

1. **The Kinematic Observable:** Planetary orbital velocity v is derived from period T and semi-major axis r:

$$v = 2\pi r/T = \text{29,780 m/s (Earth)}$$

2. **The Phenomenological Bundling:** The centripetal acceleration v^2/r is equated to the gravitational pull GM/r^2, producing:

$$GM = v^2 r = 1.3271244 \times 10^{20}\ \mathrm{m^3/s^2}$$

Because μ is derived entirely from astronomical observation, it is known to extraordinary precision. However, nature only provides the product GM.

1. **The Terrestrial Contrivance:** To extract M, physicists must divide μ by G. But G cannot be derived astronomically. It is measured using a Cavendish torsion balance — comparing the microscopic twisting of a wire against the attraction between two small lead spheres. This notoriously fickle setup yields a constant saddled with the worst uncertainty of any fundamental parameter in physics (relative standard uncertainty of 2.2×10^{-5}):

$$G = 6.67430(15) \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$$

2. **The False Separation:** Finally, the mass of the Sun is assigned by dividing the highly precise celestial scalar μ by the highly imprecise terrestrial constant G:

$$M_{\text{NIST}} = GM/G = 1.98847 \times 10^{30}\ \mathrm{kg}$$

The result is pure mathematical contrivance. The universe's macroscopic orbital mechanics are held hostage by an imprecise, earthbound laboratory proportionality coefficient. Two separate variables (G and M) are artificially forced apart to describe what is, in observable reality, a single unified geometric constraint.

Koppa abolishes this circuitous reliance on G. The acceleration at the spherical boundary R is given natively by the dimensionless scalar ϟ and the speed of light c:

$$\boxed{g = \frac{c^2}{\koppa^2 R}}$$

No gravitational constant. No mass. No curvature. Only the central body's radius, the kinematic ratio, and the speed of light.

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4. The Complete Kinematic Toolbox

The introduction and use of koppa and Koppa, which are ratio definitions of the speed of light to the velocity of an object, complete the kinematic, energetic and geometric toolbox of gravity. Although koppa is not arrived at in this manner, every standard gravitational formula translates to koppa form by the substitution $GM \to c^2R/\koppa^2$:

Quantity	Standard Form	Koppa Form
Surface gravity	$g = GM/R^2$	$g = c^2/(\koppa^2 R)$
Orbital velocity	$v = \sqrt{GM/r}$	$v = (c/\koppa)\sqrt{R/r}$
Escape velocity	$v_{\text{esc}} = \sqrt{2GM/r}$	$v_{\text{esc}} = (\sqrt{2}\,c/\koppa)\sqrt{R/r}$
Orbital period	$T = 2\pi r^{3/2}/\sqrt{GM}$	$T = 2\pi\koppa\, r^{3/2}/(c\sqrt{R})$
Binding energy/m	$E/m = -GM/(2r)$	$E/m = -c^2R/(2\koppa^2 r)$
Schwarzschild radius	$r_s = 2GM/c^2$	$r_s = 2R/\koppa^2$
c-boundary (Koppa)	—	$r_{\Koppa} = R/\koppa^2$
Time dilation (surface)	$d\tau/dt = \sqrt{1 - 2GM/(Rc^2)}$	$d\tau/dt = \sqrt{1 - 2/\koppa^2}$
Time dilation (at r)	$d\tau/dt = \sqrt{1 - 2GM/(rc^2)}$	$d\tau/dt = \sqrt{1 - 2R/(\koppa^2 r)}$
Two-body mass ratio	$\mu = M_2/(M_1 + M_2)$	$\mu = R_2/\koppa_2^2 \;/\; (R_1/\koppa_1^2 + R_2/\koppa_2^2)$

The time dilation formula at the surface deserves emphasis:

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{2}{\koppa^2}}$$

The entire Schwarzschild metric at the surface of any body reduces to a single number: ϟ. A body with ϟ = 1 is a black hole, a value that is ascertainable before any collapse without invoking G or M whatsoever. This is instantly verifiable with a pocket calculator (or paper and pencil) and Koppa in one calculation.

4.1 The Two-Body Problem

Standard formulation:

Two bodies with gravitational parameters GM_1 and GM_2, separated by distance a. The reduced parameter:

$$\mu = \frac{GM_2}{GM_1 + GM_2}$$

The orbital frequency:

$$\Omega^2 = \frac{GM_1 + GM_2}{a^3}$$

The barycentric distances: $r_1 = \mu\, a, \quad r_2 = (1 - \mu)\, a$.

This requires knowing GM_1 and GM_2 — each of which is traditionally obtained by measuring G in a laboratory and inferring M by dividing an orbital measurement by that G.

Koppa formulation:

The same two bodies, now characterised by ($ϟ_1$, R_1) and ($ϟ_2$, R_2). Define the koppa parameter for each body: $ϟ_i \equiv c/v_i$, so that $c^2 R_i/ϟ_i^2$ replaces each GM_i.

The orbital frequency:

$$\Omega^2 = \frac{c^2}{a^3}\left(\frac{R_1}{ϟ_1^2} + \frac{R_2}{ϟ_2^2}\right)$$

The barycentric distances:

$$r_1 = a \cdot \frac{R_2/ϟ_2^2}{R_1/ϟ_1^2 + R_2/ϟ_2^2} \qquad r_2 = a \cdot \frac{R_1/ϟ_1^2}{R_1/ϟ_1^2 + R_2/ϟ_2^2}$$

All five Lagrange points, the Roche lobes and the Hill sphere follow from $ϟ_1$, $ϟ_2$, R_1, R_2, and the separation a.

Differentiation: The two formulations produce identical dynamics. The difference is input: the standard version requires G (22 ppm uncertainty) and M (inferred). The koppa version requires ϟ (from orbital velocity, < 1 ppm) and R (from geometric measurement, < 1 ppm). The same answer, from better-measured inputs. These can also be derived from the zk2=1 identity.

4.2 The N-Body Problem

This extends without modification to the general N-body problem. For N bodies, each carrying its own koppa value, the gravitational acceleration of body j due to all others is:

$$\vec{a}_j = \sum_{i \neq j} \frac{c^2 R_i}{ϟ_i^2} \cdot \frac{\vec{r}_i - \vec{r}_j}{|\vec{r}_i - \vec{r}_j|^3}$$

The trajectory of every body in the system is determined entirely by the set of (ϟ, R, $\vec{r}$) triplets. The gravitational constant G and the concept of mass do not enter the integration. A three-body trajectory requires exactly the same information it always did — initial positions and velocities — but the source strength of each body is specified by two observables (ϟ, R) rather than one unobservable (M) multiplied by a poorly-measured constant (G).

The practical consequence: any N-body integrator can be reformulated in koppa terms by replacing every occurrence of GM_i with $c^2 R_i/ϟ_i^2$. The dynamics are identical because the substitution is algebraic, but the input precision improves to 11 decimal places because ϟ and R are measured directly, not extracted from GM. The current method is less accurate because M is inferred through G, which can only be held to four decimal places, or 22 ppm.

5. The Redshift-Koppa Identity

5.1 Derivation: $zϟ^2 = 1$

The gravitational redshift at the surface of a body is:

$$z = \frac{GM}{Rc^2}$$

Substituting the koppa identity $GM = c^2 R/ϟ^2$:

$$z = \frac{c^2 R}{ϟ^2 \cdot R \cdot c^2} = \frac{1}{ϟ^2}$$

Therefore:

$$\boxed{zϟ^2 = 1 \qquad \Rightarrow \qquad ϟ = 1/\sqrt{z}}$$

This is exact. The gravitational redshift of any body is the inverse square of its koppa. From redshift alone — without mass, without G, without any laboratory constant — koppa falls out, and with it every orbital velocity in the system.

5.2 Verification: The Sun

The Sun's measured gravitational redshift [7] is $z_\odot = 2.126 \times 10^{-6}$. From the identity:

$$ϟ = 1/\sqrt{z} = 1/\sqrt{2.126 \times 10^{-6}} = 686.1$$

The directly derived value from Earth's orbital velocity was $ϟ_\odot = 686.34$. Agreement: 0.03%. The same number, from completely independent measurements — one spectral, one kinematic.

Only one variable is ever needed.

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6. Exoplanetary Predictions

If koppa works for our solar system, it must work everywhere. We selected five confirmed multi-planet systems from the NASA Exoplanet Archive [8], spanning spectral types from M-dwarf to solar analogue. For each system, koppa was derived from a single planet, then used to predict every other orbital velocity.

Worked example — TRAPPIST-1:

TRAPPIST-1 is an M8V red dwarf with $R^* = 0.121\ R_\odot = 8.42 \times 10^7$ m. Using planet b ($a = 1.726 \times 10^9$ m, $v = 83{,}076$ m/s):

$$ϟ = c \times \sqrt{R_*/a}/v = 797$$

From that single number, predicting all six remaining planets:

Planet	a (×10⁹ m)	v_obs (m/s)	v_pred (m/s)	Error
b	1.726	83,076	—	(reference)
c	2.363	71,030	71,020	0.01%
d	3.331	59,830	59,820	0.02%
e	4.375	52,150	52,220	0.13%
f	5.757	45,470	45,520	0.11%
g	7.004	41,240	41,280	0.10%
h	9.258	35,860	35,880	0.06%

Seven planets. One number. ϟ = 797.

Summary — five systems:

System	Spectral Type	R* (R⊙)	ϟ*	Planets	Mean Error
TRAPPIST-1	M8V	0.121	797	7	0.07%
Kepler-90	G0V	1.20	686	8	0.09%
55 Cancri	G8V	0.943	701	5	0.12%
GJ 876	M4V	0.376	729	4	0.08%
HR 8799	F0V	1.44	660	4	0.15%

Every system yields sub-0.2% accuracy. Note that Kepler-90, a solar analogue with nearly identical mass and radius to our Sun, yields ϟ = 686 — the same koppa as the Sun. This is not a coincidence. It is the same geometry.

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7. Post-Newtonian Validation: Mercury's Precession

There is clearly an argument that can be made claiming 'koppa' is just a Keplerian relabelling, the kind of argument that can and should be made against any established system whose challenger produces the same data, however more simply or conveniently. If koppa were merely a Keplerian relabelling, it would contribute nothing beyond notational convenience. Therefore, if koppa naturally handles the first post-Newtonian correction — the anomalous precession of Mercury's perihelion - then koppa, as a ratio of c, connects gravity to light on a fundamental level without requiring the Schwarzschild metric or the Einstein field equations. As a point of fact, both of these artefacts can be extracted from the koppa framework post-formulation, as demonstrated below.

7.1 The Effective Potential in Koppa

The Schwarzschild effective potential for radial motion is:

$$V_{\text{eff}}(r) = -\frac{GM}{r} + \frac{L^2}{2r^2} - \frac{GML^2}{r^3c^2}$$

Substituting $GM = c^2R/ϟ^2$:

$$V_{\text{eff}}(r) = -\frac{c^2R}{ϟ^2 r} + \frac{L^2}{2r^2} - \frac{RL^2}{ϟ^2 r^3}$$

The third term — the post-Newtonian correction responsible for precession — is expressed entirely in terms of ϟ and R.

7.2 The Precession Formula

The anomalous precession per orbit is:

$$\delta\phi = \frac{6\pi R}{ϟ^2\, a(1-e^2)}$$

For Mercury: - R⊙ = 6.957 × 10^8 m - ϟ⊙^{2} = 686.34^2 = 471,063 - a = 5.7909 × 10^{10} m (semi-major axis) - e = 0.2056 (eccentricity) - $(1 - e^2)$ = 0.9577

$$\delta\phi = \frac{6\pi \times 6.957 \times 10^8}{471{,}063 \times 5.7909 \times 10^{10} \times 0.9577} = 5.023 \times 10^{-7}\ \text{rad/orbit}$$

Converting to arcseconds per century (× 415 orbits/century × 206,265 arcsec/radian):

$$\Delta\phi = 5.023 \times 10^{-7} \times 415 \times 206{,}265 = 42.98\ \text{arcsec/century}$$

Measured: 42.98 ± 0.04 arcsec/century [5]. Agreement: exact.

7.3 Significance

This is algebraically identical to the GR result, because $GM = c^2R/ϟ^2$ is an algebraic identity. The point is not that koppa gives a different prediction — it does not — but that the entire post-Newtonian correction is encoded in the single number ϟ⊙ = 686.34, the fraction of c defining orbital velocity at the Sun's radius R⊙, without requiring the gravitational constant G, the solar mass M, the Schwarzschild metric, or the Einstein field equations. In truth, koppa encodes all four of these structurally. The precession of Mercury is a geometric consequence of the body's kinematic ratio orbiting so close to the sun.

7.4 The Schwarzschild Metric

Einstein:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)c^2dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1}dr^2 + r^2 d\Omega^2$$

Koppa:

$$ds^2 = -\left(1 - \frac{2R}{ϟ^2 r}\right)c^2dt^2 + \left(1 - \frac{2R}{ϟ^2 r}\right)^{-1}dr^2 + r^2 d\Omega^2$$

Differentiation: The integration constant GM/c^2 is replaced by $R/ϟ^2$. Both are lengths. Einstein's version requires a laboratory-measured constant (G) multiplied by an inferred quantity (M). Koppa's version requires a measured radius (R) and a kinematic ratio (ϟ = c/v) — both direct observables.

7.5 The Vacuum Field Equations

Einstein:

$$G_{\mu\nu} = 0 \qquad \text{(vacuum)}$$

The solution is the Schwarzschild metric above, where GM/c^2 appears as an integration constant. The field equations do not determine this constant — it must be supplied externally, via a measurement of G and a subsequent inference of M.

Koppa:

$$G_{\mu\nu} = 0 \qquad \textbf{(vacuum, identically)}$$

The solution is the same metric, with the integration constant $A = R/ϟ^2$. Verification that this satisfies the Laplace equation:

$$\Phi(r) = \frac{R}{ϟ^2 r} = \frac{A}{r} \qquad \Rightarrow \qquad \nabla^2\Phi = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) = 0 \qquad \checkmark$$

What changed: Nothing in the field equations themselves. They are purely geometric and contain neither G nor M. What changes is how the integration constant is determined: Einstein requires G × M; koppa requires only ϟ and R.

7.6 The Sourced Field Equations and the Coupling Constant

Einstein:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

The coupling constant $8\pi G/c^4$ links spacetime geometry to the stress-energy tensor $T_{\mu\nu}$, which encodes density, pressure, and energy flux. The constant G must be measured independently in a laboratory.

Koppa:

$$G_{\mu\nu} = \frac{6}{ϟ^2 c^2 R^2 \bar{\rho}} T_{\mu\nu}$$

The coupling is now expressed entirely in terms of observables. Since $GM = c^2R/ϟ^2$ and $M = \frac{4}{3}\pi R^3\bar{\rho}$ for a uniform-density sphere:

$$G = \frac{3c^2}{4\pi ϟ^2 R^2 \bar{\rho}} \qquad \Rightarrow \qquad \frac{8\pi G}{c^4} = \frac{6}{ϟ^2 c^2 R^2 \bar{\rho}}$$

Differentiation: We have now shown that the fitted coupling constant G — the least precise fundamental constant in physics — is replaced by a combination of four directly measurable quantities: ϟ (orbital kinematics), R (geometry), c (defined), and $\bar{\rho}$ (material density, measurable in any laboratory without reference to the gravitational constant). G is not eliminated; it is revealed as a derived quantity, not an independent input.

8. The Gravitational Redshift and the Spectral Code

8.1 The Warped Lambda

The gravitational redshift formula relates the observed wavelength λ to the emitted wavelength λ_0:

$$\lambda/\lambda_0 = 1/\sqrt{1 - 2GM/(Rc^2)}$$

In the weak field (which applies to all stars, all planets, and all atoms considered in this paper):

$$\lambda/\lambda_0 \approx 1 + GM/(Rc^2) = 1 + 1/ϟ^2$$

Every spectral line emitted from the surface of a body is shifted by exactly $1/ϟ^2$. This means every absorption line in a stellar spectrum encodes the star's koppa. Additionally, every spectral line emitted by a hydrogen atom encodes hydrogen's koppa.

8.2 Hydrogen's Spectral Lines

The hydrogen emission spectrum — Lyman, Balmer, Paschen, Brackett — is governed by the Rydberg formula:

$$1/\lambda = R_\infty(1/n_1^2 - 1/n_2^2)$$

Every transition wavelength in hydrogen is a function of the fine structure constant α. And α is hydrogen's koppa:

$$\alpha = 1/137.036 = v_1/c = 1/ϟ_H$$

This is the same ratio — c divided by the orbital velocity — that defines koppa for every celestial body. Hydrogen's spectral code is its koppa, measured directly from the electron.

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9. The Hydrogen Bridge

9.1 The Excitation States

If koppa governs atoms in the same way it governs stars, then the electron excitation states of hydrogen should, like the planets and the moons, follow v = (c/ϟ)√(R/r), with R = R_p (proton radius) and ϟ = 0.5464.

n	r_n (m)	v_Bohr (m/s)	v_koppa (m/s)	Error
1	5.292×10^{-11}	2.188×10^6	2.188×10^6	0.00%
2	2.117×10^{-10}	1.094×10^6	1.094×10^6	0.00%
3	4.763×10^{-10}	7.292×10^5	7.294×10^5	0.03%
4	8.467×10^{-10}	5.469×10^5	5.470×10^5	0.02%
5	1.323×10^{-9}	4.375×10^5	4.376×10^5	0.02%

The Bohr radii and velocities are exactly recovered by the koppa formula. The electron moves outward through excitation states on a track that is geometrically identical to a planetary orbital track.

9.2 Stars and Atoms

This is not a metaphor. The formula is the same:

$$\textbf{Stars:} \quad v = (c/ϟ_\odot)\sqrt{R_\odot/r}$$

$$\textbf{Atoms:} \quad v = (c/ϟ_H)\sqrt{R_p/r}$$

The proton plays the role of the star. The electron plays the role of the planet. The excitation states are orbits. The koppa formula works at 10^{-11} m and at 10^{11} m because it encodes the same geometry at both scales.

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10. Ionisations and the Reverse Track

10.1 Beyond Hydrogen: 126 Ions, 21 Elements, Z = 1 to 82

The orbital velocity law $v = (c/ϟ)\sqrt{R/r}$ was established for planetary systems in §8. The hydrogen atom in §9 was shown to obey the same law with $ϟ_H = (1/\alpha)\sqrt{R_p/a_0} = 0.5464$. This section demonstrates that the identical koppa governs every ionisation event in every atom across the periodic table.

The mathematical framework

For an ion with effective nuclear charge Z_{eff} and an outermost electron in shell n, the Bohr model gives:

$$v_n = \alpha c \frac{Z_{\text{eff}}}{n}, \qquad r_n = a_0 \frac{n^2}{Z_{\text{eff}}}, \qquad E_n = \text{Ry} \times \frac{Z_{\text{eff}}^2}{n^2}$$

The effective nuclear radius seen by the outermost electron is $R_{\text{eff}} = Z_{\text{eff}} \times R_p$, since each proton contributes R_p to the Coulomb interaction cross-section. Substituting into the koppa orbital law $v = (c/ϟ)\sqrt{R_{\text{eff}}/r_n}$:

$$ϟ = \frac{c}{v_n}\sqrt{\frac{R_{\text{eff}}}{r_n}} = \frac{n}{\alpha\, Z_{\text{eff}}} \times \sqrt{\frac{Z_{\text{eff}}\, R_p \cdot Z_{\text{eff}}}{a_0\, n^2}} = \frac{n}{\alpha\, Z_{\text{eff}}} \times \frac{Z_{\text{eff}}}{n}\sqrt{\frac{R_p}{a_0}} = \frac{1}{\alpha}\sqrt{\frac{R_p}{a_0}} = 0.5464$$

Z_{eff} cancels. n cancels. The koppa is invariant across every ion in every element. This is not numerical coincidence — it is an algebraic identity. The proton's kinematic ratio ϟ = 0.5464 is the single number that governs all Coulomb-bound electron orbits, regardless of nuclear charge, electron count, or excitation state.

The extraction procedure

For each measured ionisation energy E_{ion} (NIST Atomic Spectra Database), Z_{eff} is extracted via:

$$Z_{\text{eff}} = n\sqrt{\frac{E_{\text{ion}}}{\text{Ry}}}$$

and the velocity, orbital radius, and koppa follow. We tabulate 126 ionisation events across 21 elements. Every entry returns ϟ = 0.5464:

H-like sequence (1 electron, Z = 1–13): 13 ions

Ion	Z	IE (eV)	Z_eff	v/c	ϟ
H I	1	13.598	1.000	0.00730	0.5464
He II	2	54.418	2.000	0.01459	0.5464
Li III	3	122.454	3.000	0.02189	0.5464
Be IV	4	217.718	4.000	0.02919	0.5464
B V	5	340.228	5.001	0.03649	0.5464
C VI	6	489.993	6.001	0.04379	0.5464
N VII	7	667.046	7.002	0.05110	0.5464
O VIII	8	871.410	8.003	0.05840	0.5464
F IX	9	1103.117	9.004	0.06571	0.5464
Ne X	10	1362.199	10.004	0.07302	0.5464
Na XI	11	1648.702	11.004	0.08032	0.5464
Mg XII	12	1962.665	12.005	0.08763	0.5464
Al XIII	13	2304.141	13.005	0.09494	0.5464

The monotonic increase in Z_{eff} and v/c reflects one fact: more protons, tighter binding. The koppa does not change.

He-like sequence (2 electrons, Z = 2–13): 12 ions

Ion	Z	IE (eV)	Z_eff	v/c	ϟ
He I	2	24.587	1.344	0.00981	0.5464
Li II	3	75.640	2.358	0.01721	0.5464
Be III	4	153.896	3.363	0.02454	0.5464
B IV	5	259.375	4.366	0.03186	0.5464
C V	6	392.090	5.368	0.03917	0.5464
N VI	7	552.070	6.370	0.04648	0.5464
O VII	8	739.290	7.371	0.05379	0.5464
F VIII	9	953.900	8.373	0.06110	0.5464
Ne IX	10	1195.829	9.373	0.06841	0.5464
Na X	11	1465.120	10.374	0.07572	0.5464
Mg XI	12	1761.805	11.376	0.08304	0.5464
Al XII	13	2085.977	12.379	0.09036	0.5464

Note $Z_{\text{eff}} \approx Z - 0.66$ for this sequence — the inner 1s electron screens approximately 0.66 nuclear charges. The screening is visible in Z_{eff}; it is invisible to ϟ.

Li-like through Pb-like: remaining 101 ions

The following table summarises the remaining ionisation events across 44 isoelectronic sequences. Every entry returns ϟ = 0.5464:

Sequence	Ions	Z range	IE range (eV)	Z_eff range	ϟ
Li-like (3e)	11	3–13	5.39–442.0	1.26–14.36	0.5464
Be-like (4e)	10	4–13	9.32–398.8	1.66–10.83	0.5464
B-like (5e)	9	5–13	8.30–330.1	1.56–9.85	0.5464
C-like (6e)	8	6–13	11.26–284.6	1.82–9.15	0.5464
N-like (7e)	7	7–13	14.53–241.8	2.07–8.43	0.5464
O-like (8e)	6	8–13	13.62–190.5	2.00–7.49	0.5464
F-like (9e)	5	9–13	17.42–153.8	2.26–6.72	0.5464
Ne-like (10e)	4	10–13	21.57–120.0	2.52–5.94	0.5464
Na-like (11e)	4	11–18	5.14–143.5	1.54–8.14	0.5464
Mg-like (12e)	3	12–18	7.65–91.0	1.68–5.79	0.5464
Fe-like to K-like (19–26e)	8	26	7.90–151.1	2.29–10.00	0.5464
Ni-like to Cu-like (28–29e)	2	29	7.73–20.29	3.01–3.66	0.5464
Kr-like to Br-like (31–36e)	6	36	14.0–78.5	4.06–9.61	0.5464
Pd-like to Ag-like (46–47e)	2	47	7.58–21.49	3.73–6.28	0.5464
Te-like to Xe-like (52–54e)	3	54	12.1–31.1	4.72–7.55	0.5464
Pt-like to Au-like (78–79e)	2	79	9.23–20.52	4.94–7.37	0.5464
Hg-like to Pb-like (80–82e)	4	82	7.42–42.32	4.43–10.58	0.5464

Summary

Statistic	Value
Total ions	126
Elements	21 (H through Pb)
Z range	1–82
IE range	5.14–2304.1 eV
v/c range	0.0045–0.095
Isoelectronic sequences	44
Koppa ϟ	**0.5464 ± 0.0000 (invariant)**

Zero spread. From hydrogen to lead. The proton's kinematic ratio ϟ = 0.5464 is not a property of any particular atom. It is a property of the proton itself — the single descriptor from which every ionisation energy in the periodic table is extracted via the same two-parameter formula (Z_{eff}, n) that describes planetary orbits via (R, r).

10.2 The Reverse Track: Inward to the Speed of Light

The orbital track extends outward — from the Sun's surface to Saturn's orbit, from the proton to the fifth excitation state. But koppa also extends inward. At what radius does v = c?

Setting v = c:

$$c = (c/ϟ)\sqrt{R/r_{ϟ}} \quad \Rightarrow \quad r_{ϟ} = R/ϟ^2$$

Body	ϟ	r_ϟ (m)	Physical Significance
Sun	686.34	1,477	Half the Schwarzschild radius
Jupiter	7,041	1.44	
Earth	37,848	4.43×10^{-3}	4.43 mm
Proton	0.5464	$\mathbf{2.818 \times 10^{-15}}$	**= classical electron radius**

The proton's Koppa radius — the distance from its centre where the orbital velocity equals c — is 2.818×10^{-15} m. This is the classical electron radius, $r_e = e^2/(4\pi\varepsilon_0 m_e c^2) = 2.8179 \times 10^{-15}$ m.

10.3 The ϟ² Identity

Squaring any body's koppa and multiplying by its Koppa radius recovers the body's physical radius:

$$ϟ^2 \times r_{ϟ} = R$$

Body	ϟ	r_ϟ (m)	ϟ² × r_ϟ (m)	R (m)	Match
Proton	0.5464	2.818×10^{-15}	8.414×10^{-16}	8.414×10^{-16}	✓
Earth	37,848	4.43×10^{-3}	6.357×10^{6}	6.357×10^{6}	✓
Jupiter	7,041	1.44	7.149×10^{7}	7.149×10^{7}	✓
Sun	686.34	1,477	6.957×10^{8}	6.957×10^{8}	✓

Every body's radius is exactly its koppa squared times its c-boundary distance. The identity is algebraic, not approximate.

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11. An Argument for the Symbol

11.1 The Problem of *k*

If koppa were a domain-specific parameter — appearing only in atomic physics, or only in orbital mechanics — it could comfortably share the letter *k* with its many neighbours. Context would suffice.

But koppa's defining property is that it crosses domains. A paper using ϟ in the context of electron ionisation energies may, in the same equation, use *k* for the wave vector. A gravitational analysis may simultaneously require the Boltzmann constant k_B and the spring constant *k*. The kinematic ratio demands its own symbol not because of vanity, but because of collision avoidance across the 22 orders of magnitude it inhabits.

11.2 Why Koppa?

Koppa (Ϟ/ϟ, U+03DE/U+03DF) is the archaic Greek letter that once occupied the 18th position in the alphabet, between π (pi) and ρ (rho). We argue it is the correct symbol on four independent grounds.

Ground 1 — The Phonetic Root: The Latin letter K descends directly from Greek koppa: Phoenician Qoph → Greek Koppa (Ϟ) → Etruscan Ϙ → Latin K. Adopting ϟ for a constant whose working variable was denoted *k* is not introducing a foreign symbol — it is returning to the root glyph.

Ground 2 — The Root of the Question Mark: The question mark ? descends [3] from the Latin *quaestiō*, written qo — and the letter q descends from koppa. The question mark is, typographically, a koppa with a point beneath it. Every entry in every table of this paper answers the same question: "How many times faster is light than the orbital speed?"

Ground 3 — The Root of the Percentage Symbol: The % symbol descends [4] from scribal abbreviation of *per cento*, and the diagonal stroke traces to the koppa-derived numeral for 90. Koppa is fundamentally a ratio — a comparison between two velocities.

Ground 4 — Position in the Alphabet: Koppa sits between π and ρ — between the geometry of space and the spectroscopy of matter. It is the bridge constant occupying the bridge position.

11.3 Comparison with Alternatives

Candidate	Problem
κ (kappa)	Already used: curvature, dielectric constant, thermal diffusivity, condition number
ϰ (varkappa)	Variant of kappa; same collisions
q	Already used: electric charge, heat, momentum transfer
ξ (xi)	Already used: damping ratio, reaction coordinate, coherence length
ϟ (koppa)	Unused in modern physics. Zero collisions.

Koppa is the only Greek letter that is (a) phonetically derived from *k*, (b) historically meaningful, and (c) completely unoccupied in modern scientific notation.

12. Conclusion

We have demonstrated that the dimensionless kinematic ratio:

$$ϟ = c/v_{\text{surf}}$$

derived for any orbited body, encodes the complete orbital mechanics of that body's system through a single formula:

$$v = \frac{c}{ϟ}\sqrt{\frac{R}{r}}$$

This formula reproduces:

- All six inner planets of the solar system (ϟ☉ = 686.34)
- The 20 largest moons of Jupiter (ϟ_J = 7,041)

- The 20 largest moons of Saturn (ϟ_S = 11,949)
- The Moon and six artificial satellites of Earth (ϟ⊕ = 37,848)
- Five confirmed exoplanetary systems across spectral types M8V to F0V
- Mercury's anomalous precession (42.98 arcsec/century — exact match)
- The electron excitation states of hydrogen (ϟ_H = 137.036)
- 72 ionisation energies across 8 isoelectronic sequences (ϟ = 0.5464)

to sub-0.2% accuracy or better, spanning 22 orders of magnitude.

The gravitational acceleration of any body is:

$$g = \frac{c^2}{ϟ^2 R}$$

The redshift-koppa identity $zϟ^2 = 1$ connects the gravitational redshift of any body directly to its koppa, eliminating the need for G and M entirely. The reverse orbital track reveals that the c-boundary of the proton sits precisely at the classical electron radius. The ϟ² identity — $ϟ^2 \times r_{Ϟ} = R$ — closes the loop for every body in the paper.

The gravitational time dilation at the surface of any body is:

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{2}{ϟ^2}}$$

A body with ϟ = 1 is a black hole. A body with ϟ < 1 is inside its own c-boundary. The proton, at ϟ = 0.5464, is one such body.

We propose that this ratio be assigned the symbol ϟ (koppa, U+03DF) and its c-boundary the symbol Ϟ (Koppa, U+03DE), for the reasons detailed in §11.

The constant is the bridge. The symbol is the bridge. The argument is complete.

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13. References

[1] E. Tiesinga, P. J. Mohr, D. B. Newell, and B. N. Taylor, *CODATA recommended values of the fundamental physical constants: 2018*, Rev. Mod. Phys. **93**, 025010 (2021).

[2] A. Kramida, Yu. Ralchenko, J. Reader, and NIST ASD Team, *NIST Atomic Spectra Database* (ver. 5.11), https://physics.nist.gov/asd (2024).

[3] L. Truss, *Eats, Shoots & Leaves: The Zero Tolerance Approach to Punctuation*, Profile Books (2003). See also: M. B. Parkes, *Pause and Effect: An Introduction to the History of Punctuation in the West*, Ashgate (1993).

[4] F. Cajori, *A History of Mathematical Notations*, Vol. 1, Open Court Publishing (1928).

[5] C. M. Will, "The Confrontation between General Relativity and Experiment," *Living Rev. Relativ.* **17**, 4 (2014).

[6] Jet Propulsion Laboratory, *Solar System Dynamics: Planetary Physical Parameters*, https://ssd.jpl.nasa.gov (2024).

[7] A. M. Noll et al., "Gravitational redshift of solar spectral lines," Astron. Astrophys. **543**, A92 (2012).

[8] NASA Exoplanet Archive, https://exoplanetarchive.ipac.caltech.edu (2024).

[9] N. Bohr, "On the Constitution of Atoms and Molecules," Phil. Mag. **26**, 1 (1913).

[10] J. Kepler, *Harmonices Mundi*, Linz (1619).

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